Solution of the spatial kinetic equations using the expansion in pseudo-harmonics: case 2D

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Recently was demonstrated that the steady-state fixed source problems, usually represented by a non-homogeneous linear system of equations, can be solved by the method of the pseudo-harmonics [1]. In this case the fixed source problem was solved considering that the neutron flux can be expanded in pseudo-harmonics. The method of the pseudo-harmonics was just summarized in the determination of the coefficients of the expansion of the neutron flux, substituting like this a scheme of numeric inversion of the matrix of the linear system, usually employee in this problem type.
The present work has as objective of studying the possibility of applyinh the method of the pseudo-harmonic in the solution of the linear system, which appears starting from the discretization of the equations of the spatial kinetic.
The method was tested for the cases 1D and 2D.

Assuming that the solution of the equation the spatial kinetic is given by the following expansion:

$$\begin{pmatrix} \underset{\sim}{\phi}_1 \\ \underset{\sim}{\phi}_2 \end{pmatrix}^{(n+1)} = \sum_{i=1}^{NP} \left(\alpha_{i,1}^{(n+1)} \begin{pmatrix} \underset{\sim}{\omega}_{1,i} \\ \underset{\sim}{0} \end{pmatrix}^{(n+1)} + \alpha_{i,2}^{(n+1)} \begin{pmatrix} \underset{\sim}{0} \\ \underset{\sim}{\omega}_{2,i} \end{pmatrix}^{(n+1)} \right) \qquad (1)$$

where $\underset{\sim}{\omega}_{g,j}$ is the pseudo-harmonic, obtained starting from the following eigenvalues problem

$$\left[T_{gg} \right] \underset{\sim}{\omega}_{g,j}^{(n+1)} = \lambda_{g,j}^{(n+1)} \underset{\sim}{\omega}_{g,j}^{(n+1)} \qquad (2)$$

the equation of the spatial kinetic is had [2]:

$$\begin{bmatrix} T_{11} & 0 \\ 0 & T_{22} \end{bmatrix} \begin{pmatrix} \underset{\sim}{\phi}_1 \\ \underset{\sim}{\phi}_2 \end{pmatrix}^{(n+1)} = \sum_{i=1}^{NP} \left(\alpha_{i,1}^{(n+1)} \begin{pmatrix} \lambda_{1,i} \underset{\sim}{\omega}_{1,i} \\ \underset{\sim}{0} \end{pmatrix}^{(n+1)} + \alpha_{i,2}^{(n+1)} \begin{pmatrix} \underset{\sim}{0} \\ \lambda_{2,i} \underset{\sim}{\omega}_{2,i} \end{pmatrix}^{(n+1)} \right) = \begin{pmatrix} \underset{\sim}{q}_1 \\ \underset{\sim}{q}_2 \end{pmatrix}^{(n)} . \qquad (3)$$

Multiplying the eq. (3) for $\underset{\sim}{\omega}_{g,j}^{T}$, integrating the resulting equation, and using the property

$$\left\langle \underset{\sim}{\omega}_{g,j}^{T} \underset{\sim}{\omega}_{g,i} \right\rangle^{(n+1)} = 0 , \text{ for } j \neq i ,$$

are obtained the coefficients of the expansion in (1):

$$\alpha_{i,g}^{(n+1)} = \frac{\left\langle \underset{\sim}{\omega}_{g,i}^{T} \underset{\sim}{q}_g \right\rangle^{(n+1)}}{\lambda_{g,i} \left\langle \underset{\sim}{\omega}_{g,i}^{T} \underset{\sim}{\omega}_{g,i} \right\rangle^{(n+1)}} , \text{ for } g = 1,2 . \qquad (4)$$

To test the proposed method is considered the 2-D TWIGL Seed-Blanket Reactor Benchmark. The Figure 1 displays the comparison of the normalized power. In this figure is also shown the normalized power obtained by the method proposed without using an extrapolation [1], being this indicated by an asterisk.

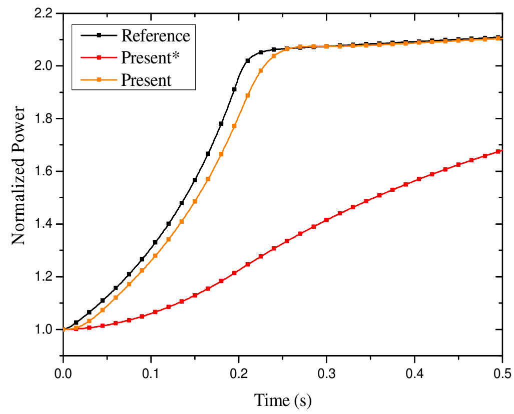

Figure 1. Variation of the Normalized Power.

The results which were generated by the present method had a good accuracy when compared with the reference results, mainly when the suggested extrapolation was applied. In 2-D TWIGL Benchmark, although it presented a maximum percentile relative deviation of 7.6%, the value obtained in the end of the transient for the percentile relative deviation, it was lesser than 1%, showing the validity of the considered method.

References

[1] Z. R. de Lima, M. L. Moreira, F. C. da Silva, A. C. M. Alvim, “Solution of the Spatial Kinetic Equations Using the Expansion in Pseudo-Harmonics: Cases 1D and 2D”. Belo-Horizonte, INAC 2011.