Application of the method of the finite element in the steady-state diffusion equation

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The aim of this paper is to present a finite element formulation for the steady-state neutron diffusion and the potential improvement of the neutron flux distribution when the adaptive remeshing is used. The error indicator used in the adaptive remeshing is based on the continuity of the neutron current at the inter-finite element interface. Few papers treat this problem using finite element method. The majority of authors (or programs) prefer to use the finite difference [1]. More recent work brought the nodal method to the calculation of steady state diffusion and space-time neutron kinetics since the finite difference is prohibitively expensive in 3D reactors. This paper adds to the nodal method an improvement in the discretization by the use of an adaptive remeshing.

Neutron Diffusion Equation Integral Formalism

The neutron diffusion differential equation can be presented in an integral formulation (weak form) that is suitable for the finite element method discretization. Multiplying the differential equation by a weight function and integrating it all over the domain :

$$\int_{\Omega}(\nabla\cdot\mathbf{p}\nabla\mathbf{u}+\mathbf{q}\mathbf{u})\mathbf{w}d\Omega=\int_{\Omega}\mathbf{f}\mathbf{w}d\Omega \qquad (1)$$

Using the derivative by parts and the Green theorem this equation can be written by:

$$\int_{\Omega}(\mathbf{p}\nabla\mathbf{u}\cdot\nabla\mathbf{w}+\mathbf{q}\mathbf{u}\mathbf{w})d\Omega= \int_{\Gamma}\mathbf{p}\nabla\mathbf{u}\mathbf{w}\cdot\mathbf{n}_{\Gamma}d\Gamma+\int_{\Omega}\mathbf{f}\mathbf{w}d\Omega \qquad (2)$$

The geometry discretization is done using the finite element method. Supposing to define a sub-domain Ω^e represented by the finite element, the entire domain Ω can be approximated by $\Omega=\sum_{e=1}^{ne}\Omega^e$. Using here the linear triangular element (2D) or the linear tetrahedral element (3D) the geometry $\mathbf{x}^e$, inside each element e, can be represented by:

$$\mathbf{x}^e=\sum_{n=1}^{nn}\mathbf{N}_n\mathbf{x}_n^e \qquad (3)$$

where nn is the number of nodes of the finite element (=3 for the linear triangle and =4 for the linear tetrahedra), $\mathbf{N}_n$ is the matrix of the linear interpolation functions at node n and $\mathbf{x}_n^e$ is the coordinates of the node n of the finite element e.

To test the method is considered the PWR Korean reactor named ULCHIN 1. The Figure 1 displays thermal neutron flux for the pin to pin using the refined mesh.

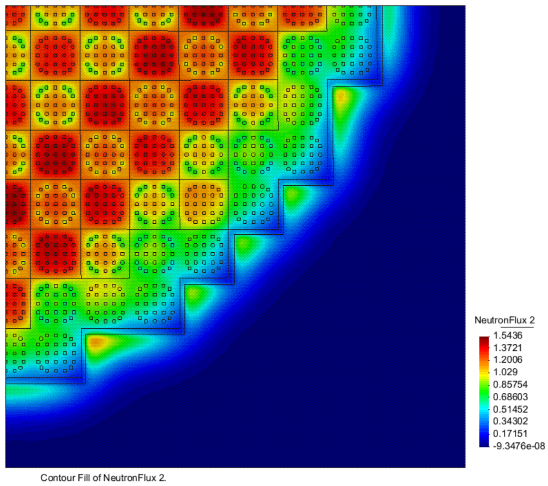

Figure 1. Thermal flux for the pin to pin ULCHIN1.

Whilst a lot of researchers use methods like nodal, finite differences, and so on; the results presented in this paper show that the finite element method can be considered a helpful tool to calculate the multiplication factor and the neutron flux even in a pin-to-pin calculations. The use of an adaptive remeshing shows that a good improvement is obtained in calculating these variables.

References

[1] L. M. Grossman, J. P. Hennart, "Nod Meth for Space Time Neutron Kinetics," Progress in Nuclear Energy, 49, pp.181-216 (2007).