

# Development of a software for perform shakedown analysis using the basis reduction technique

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The structural analysis and the safety assessment of advanced nuclear reactors will be performed by the Design by Analysis (DBA) route: a modern trend in international mechanical engineering design codes for pressure vessel and piping. However, to apply the DBA to structures under variable mechanical and thermal loads, it is necessary to assure that plastic collapse modes, low cycle fatigue and incremental collapse (with instantaneous plastic collapse as a particular case), will be precluded. The tool available to achieve this target is the shakedown theory. Unfortunately, the practical numerical applications of the shakedown theory, even in simple cases, result in very large nonlinear optimization problems with nonlinear constraints. To face real problems in an industrial level, it is necessary the development of precise, robust and efficient algorithms to solve the problem in finite dimension.

This has been a recent achievement in other countries, however resulting software are proprietary. This work seeks to endow the Nuclear Engineering Institute (IEN) with a software to perform this kind of analysis. In a first stage, for elastic ideally-plastic materials, using 3D quadratic irreducible finite elements instead of usual, but more complex mixed elements. In a second step, we will be considering more realistic properties of materials as limited kinematic hardening, using internal thermodynamical variables to model kinematic hardening materials.

As preliminary step we need a 3D distribution of temperature through the solid and it is necessary to know the corresponding generated mechanical stresses. Then it was needed to develop the software DOMCALT for heat transfer using 10 nodes tetrahedron finite elements, for isotropic and anisotropic materials, with or without internal heat generation and for steady state or transient regimes, with conduction (Neumann) and convection (Robin) boundary conditions. This software is already operational and it can be used for other objectives and thus a detailed documentation will be presented later in a IEN Technical Note as a spinoff of this work. To obtain in finite dimension, mechanical stresses due to temperature and support displacements necessary as input for the shakedown

analysis, an elastic analysis software DOMSOLID is now being developed for n materials and loadings and for any type cinematic element (including the 10 nodes quadratic tetrahedron). After being terminated and tested, the theory and the detailed documentation will be also showed in other IEN Technical Note and it will be also a spinoff for this research. To solve the shakedown problem we need to know the amplifier coefficient and the residual stress field, both obeying the static variational principle. Considering the fact that the residual stress belongs to the kernel of equilibrium operator, we can solve an equivalent problem in a subspace with a very smaller dimension using the reduced basis technique [1-3], eliminating the equilibrium constraint. At each step of algorithm an appropriate residual base needs to be found. For this aim, Shen and Zhang proposed to perform equilibrium iterations during an incremental elasto-plastic step. Usually, this can be done using an implicit Euler algorithm but it is necessary to solve a nonlinear system by Newton-Raphson. Nevertheless, it can be accomplished in a very simply way, for Mises materials in 3D, using a radial return map algorithm. The theory for this steps is already detailed at finite dimension level. The last step will be used the Box Complex Method [2] to solve the nonlinear optimization problem in the subspace, this theory is now under development.

## References

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