Using the PCA method in classical point kinetics and modified point kinetics for transient reactivity step

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The objective of this work is to use the numerical method known as PCA - "Piecewise Constant Approximation" in the solution of the classical point kinetics equations for six neutron precursor groups. The time-dependent reactivity function is approximated by piecewise constant functions and the resulting first-order ordinary differential equation system is solved exactly over each time step. Numerical simulations with step reactivity functions are made and the solutions are compared with results given in the published literature. The method is later extended to the so called modified neutron point kinetics, which has the time derivative of the neutron current not negligible.

The PCA method consists of approximating the reactivity and source term functions, present in the equations of classical and modified point kinetics, by constant functions by parts in each time partition [1]:

$$\rho(t) \approx \rho\left(\frac{t_i + t_{i+1}}{2}\right) = \rho_i, \quad t_i \leq t \leq t_{i+1} \qquad (1)$$

$$F(t) \approx F\left(\frac{t_i + t_{i+1}}{2}\right) = F_i, \quad t_i \leq t \leq t_{i+1} \qquad (2)$$

To implement the method, the intrinsic function of the Wolfram Mathematica [2] software was used. The results using the PCA method to obtain numerical solutions of the modified point kinetics equations are presented. Solutions with step functions are compared with the finite Differences Method (FD) and Classical Point Kinetics (CPK). The kinetic parameters and other data used in the simulations are listed in Table 1. Tables 2 and 3 shows the neutron density calculations of the modified point kinetics using different methods for a step-type reactivity equal to 300 pcm and 700 pcm, respectively.

For the insertion of reactivity of 300 pcm, there is no great divergence from classic point kinetics to modified point kinetics, both for transport frequency ($1/\tau$) 10^3 and for 10^4 s^{-1}.

Table 1 - Modified kinetics parameters for six groups of precursors

Parameter	Value
Λ	0.00002 s
$\beta = \beta_1 + \beta_2 + \cdots + \beta_6$	0.007
Σ_a	0.1718 cm^{-1}
D	10 cm
υ	3×10^5 cm/s
fa	51540 s^{-1}
τ	10^{-3} / 10^{-4} s

Table 2 - Density calculations of the modified point kinetics using different methods for a step-type reactivity equal to 300pcm.

Method	t = 1.0 s	t = 10 s	t = 20 s
PCA (h=0,1s) τ= 10^{-4}s	2.20888	8.01333	28.2654
PCA (h=0,1s) τ= 10^{-3}s	2.20012	7.9604	27.9789
FD, τ= 10^{-4}s	2.208891	8.013279	28.26504
FD, τ= 10^{-3}s	2.200393	7.961013	27.98072
CPK - Exact	2.2098	8.0192	28.297

Table 3 - Density calculations of the modified point kinetics using different methods for a step-type reactivity equal to 700 pcm.

Method	t = 0.01 s	t = 0.5 s	t = 2 s
PCA (h=0,1s) τ= 10^{-4}s	3.94994	2975.14	2.49744×10^{10}
PCA (h=0,1s) τ= 10^{-3}s	2.21233	270.71	5.62973×10^{6}
FD, τ= 10^{-4}s	3.954635	2974.806	2.495043×10^{10}
FD, τ= 10^{-3}s	2.291743	271.8574	5.651582×10^{6}
CPK - Exact	4.5088	5345.9	2.0591×10^{11}

As for 700 pcm and in relation to the classic point kinetics, it presents a faster growth of the neutron density, reaching orders of magnitude greater than the modified point kinetics.

References

[1] MARTINS, B. N. Aplicações do Método de Aproximação por Funções Constantes por Partes na Cinética Pontual Clássica e na Cinética Pontual Modificada. Dissertação de Mestrado, IEN/CNEN, 2019.

[2] SOFTWARE MATHEMATICA. available on the website https://www.wolfram.com/mathematica