Implementation of a numeric algorithm for the solution of the generalized Christoffel equation referred to orthotropic materials under plane stress state

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Extensive studies carried out in the last decades on the propagation of ultrasonic waves in solids led to the development of nondestructive techniques for the assessment of the safety and integrity of industrial structures and components. The interest in the application of ultrasound techniques for stress measurement, for example, comes from the measurable change in the speed of the ultrasonic elastic waves in the presence of a stress field, a phenomenon known as acoustoelastic effect. An acoustoelastic theory provides an attractive way of non-destructively measuring the average stress along the wave's path. The study of the propagation of ultrasonic waves in homogenous anisotropic bodies under stress leads to a nonlinear eigenvalue problem given by the generalized Christoffel equation. The nonlinearity characteristic of the problem derives from the interdependence between the material's effective elastic constants and the acting stresses. The experimental measurement of stresses using ultrasound techniques is an inverse problem of acoustoelasticity [1-3].

This work presents the implementation of numeric algorithm based on the Degtyar and Rokhlin (1995) method for solution of the generalized Christoffel equation, formulated according to Man and Lu (1987) acoustoelastic theory. The algorithm developed is currently applicable to orthotropic materials under plane stress states in which the direction of the principal stresses are aligned with the direction of the material axes of symmetry. The solution of the generalized Christoffel equation poses difficulties of practical and theoretical orders. The stability and precision of the developed algorithm, as well as the influence of the uncertainties in the experimental measurement of the velocities of the ultrasonic waves were then investigated. Synthetic velocity data for waves of oblique incidence in a plate subjected to a plane stress state was generated by the solution of the generalized Christoffel equation in direct form in order to illustrate the application of the developed algorithm.

The results indicated that the algorithm seems to be robust enough in the sense that uncertainties in the input data (up to a certain degree) can handled, and that precise measurement of the velocities of the longitudinal or shear waves propagating in the material is the overriding influential factor for the reconstruction of the stress dependent elastic constant and the stresses.

The main objective of this line of research work is to make available in the country a new numerical tool to support the application of ultrasonic waves for experimental stress analysis.

References

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